LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2018**

MT 5508/MT 5502 – LINEAR ALGEBRA

Date: 01-11-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

ANSWER ALL THE QUESTIONS **(10×2=20)**

1. Define dimension of a vector space.
2. Give an example for a vector space homomorphism.
3. Define basis for a vector space.
4. Prove that any $n+1$ vectors in F^n are linearly independent.
5. Let R^3 be the inner product over R under standard inner product. Find the norm of $(3,0,4)$.
6. Define a dual space.
7. Let $T \in A(\text{V})$ and $\lambda \in F$. If λ is an eigen value of T, prove that $\lambda I - T$ is singular.
8. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
9. Define unitary linear transformation.
10. Define rank of a matrix.

PART – B

ANSWER ANY FIVE QUESTIONS **(5×8=40)**

11. Let V be a vector space and suppose that one basis has n elements and another basis has m elements. Then prove that $m = n$.
12. Prove that the intersection of two sub-spaces of a vector space V is a subspace of V.
13. Express the vector $(1,-2,5)$ as a linear combination of the vectors $(1,1,1)$, $(1,2,3)$ and $(2,-1,1)$ in R^3.
14. Prove that the vectors $(1,0,0)$, $(1,1,0)$ and $(1,1,1)$ form a basis of R^3, where R is the field of real numbers.
15. State and prove Schwarz inequality.
16. If $S,T \in A(\text{V})$ then prove that (i) $\text{rank(ST)} \le \text{rank of T}$ (ii) $\text{rank(ST)} \le \text{rank of S}$.
17. Show that any square matrix can be expressed as a sum of a symmetric matrix and a skew symmetric matrix.
18. Show that the system of equations $x_1 + 2x_2 + x_3 = 11$, $4x_1 + 6x_2 + 5x_3 = 8$ and $2x_1 + 2x_2 + 3x_3 = 19$ is inconsistent .

PART – C

ANSWER ANY TWO QUESTIONS **(2×20=40)**

19. (a) If S and T are subsets of a vector space V over F, then prove that

    (i) S is a subspace of V if and only if $L(S) = S$.

(ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.

(iii) $L(L(S)) = L(S)$.

(iv) $L(S \cup T) = L(S) + L(T)$.

(b) If V is a vector space of finite dimension and w is a subspace of V is a subspace of V, then prove that $\dim V/W = \dim V - \dim W$. **(10+10)**

20. a) If U and V are vector spaces over F, and if T is a homomorphism of U onto V with kernel W, then prove that $U/W \cong V$.

b) Prove that $T \in A(V)$ is invertible if and only if whenever $v_1, v_2, ...v_n$ are in V and linearly independent, then $T(v_1), T(v_2), ...T(v_n)$ are also linearly independent. **(12+8)**

21. Apply the Gram – Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors $(1,1,0,1)$, $(1,-2,0,0)$ and $(1,0,-1,2)$.

22. a) Prove that the linear transformation Ton V is unitary of and only if it takes an orthonormal basis of V onto an orthonormal basis of V.

b) Prove that for any $m \times n$ matrix over a field F, the row rank and column rank are equal. **(12+8)**